

2.6 Solving Two-Step Equations

Essential Question In a two-step equation, which step should you do first?

1 EXAMPLE: Solving a Two-Step Equation

Use algebra tiles to model and solve $2x - 3 = -5$.

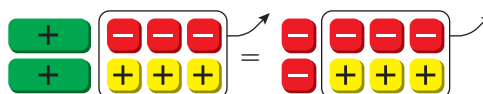
Model the equation $2x - 3 = -5$.



Remove the three red tiles on the left side by adding three yellow tiles to each side.



Remove the three "zero pairs" from each side.



Because there are two green tiles, divide the red tiles into two equal groups.



Keep one of the groups. This shows the value of x .



So, $x = -1$.

2 EXAMPLE: The Math Behind the Tiles

Solve $2x - 3 = -5$ without using algebra tiles. Describe each step. Which step is first, adding 3 to each side or dividing each side by 2?

Use the steps in Example 1 as a guide.

$$\rightarrow 2x - 3 = -5 \quad \text{Write the equation.}$$

$$\rightarrow 2x - 3 + 3 = -5 + 3 \quad \text{Add 3 to each side.}$$

$$\rightarrow 2x = -2 \quad \text{Simplify.}$$

$$\rightarrow \frac{2x}{2} = \frac{-2}{2} \quad \text{Divide each side by 2.}$$

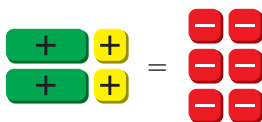
$$\rightarrow x = -1 \quad \text{Simplify.}$$

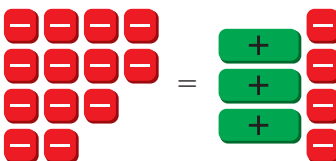
So, $x = -1$. Adding 3 to each side is the first step.

3 ACTIVITY: Solving Equations Using Algebra Tiles

Work with a partner.

- Write an equation shown by the algebra tiles.
- Use algebra tiles to model and solve the equation.
- Check your answer by solving the equation without using algebra tiles.

a. 

b. 

4 ACTIVITY: Working Backwards

Work with a partner.

- a. **Sample:** Your friend pauses a video game to get a drink. You continue the game. You double the score by saving a princess. Then you lose 75 points because you do not collect the treasure. You finish the game with -25 points. How many points did you start with?

One way to solve the problem is to work backwards. To do this, start with the end result and retrace the events.

You have -25 points at the end of the game. -25

You lost 75 points for not collecting the treasure, so add 75 to -25 . $-25 + 75 = 50$

You doubled your score for saving the princess, so find half of 50. $50 \div 2 = 25$

∴ So, you started the game with 25 points.

- b. You triple your account balance by making a deposit. Then you withdraw \$127.32 to buy groceries. Your account is now overdrawn by \$10.56. By working backwards, find your account balance before you made the deposit.

What Is Your Answer?

5. **IN YOUR OWN WORDS** In a two-step equation, which step should you do first? Give four examples.
6. Solve the equation $2x - 75 = -25$. How do your steps compare with the strategy of working backwards in Activity 4?

Practice 

Use what you learned about solving two-step equations to complete Exercises 6–11 on page 86.

EXAMPLE 1 Solving a Two-Step Equation

 Solve $-3x + 5 = 2$. Check your solution.

$$-3x + 5 = 2$$

Write the equation.

$$\underline{-5} \quad \underline{-5}$$

Subtract 5 from each side.

$$-3x = -3$$

Simplify.

$$\frac{-3x}{-3} = \frac{-3}{-3}$$

 Divide each side by -3 .

$$x = 1$$

Simplify.

Check

$$-3x + 5 = 2$$

$$-3(1) + 5 \stackrel{?}{=} 2$$

$$-3 + 5 \stackrel{?}{=} 2$$

$$2 = 2 \quad \checkmark$$

 So, the solution is $x = 1$.

 On Your Own

Solve the equation. Check your solution.

Now You're Ready
Exercises 6–17

1. $2x + 12 = 4$

2. $-5c + 9 = -16$

3. $3(x - 4) = 9$

EXAMPLE 2 Solving a Two-Step Equation

Solve $\frac{x}{8} - \frac{1}{2} = -\frac{7}{2}$.

$$\frac{x}{8} - \frac{1}{2} = -\frac{7}{2}$$

Write the equation.

$$\underline{+\frac{1}{2}} \quad \underline{+\frac{1}{2}}$$

 Add $\frac{1}{2}$ to each side.

$$\frac{x}{8} = -3$$

Simplify.

$$8 \cdot \frac{x}{8} = 8 \cdot (-3)$$

Multiply each side by 8.

$$x = -24$$

Simplify.

 So, the solution is $x = -24$.

 On Your Own

Solve the equation. Check your solution.

Now You're Ready
Exercises 20–25

4. $\frac{m}{2} + 6 = 10$

5. $-\frac{z}{3} + 5 = 9$

6. $\frac{2}{5} + 4a = -\frac{6}{5}$

Study Tip

You can simplify the equation in Example 2 before solving. Multiply each side by the LCD of the fractions, 8.

$$\frac{x}{8} - \frac{1}{2} = -\frac{7}{2}$$

$$x - 4 = -28$$

$$x = -24$$

EXAMPLE 3 Combining Like Terms Before Solving

Solve $3y - 8y = 25$.

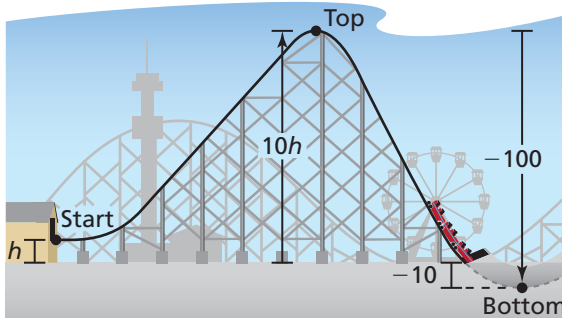
$$3y - 8y = 25 \quad \text{Write the equation.}$$

$$-5y = 25 \quad \text{Combine like terms.}$$

$$y = -5 \quad \text{Divide each side by } -5.$$

So, the solution is $y = -5$.

EXAMPLE 4 Real-Life Application



The height at the top of a roller coaster hill is 10 times the height h of the starting point. The height decreases 100 feet from the top to the bottom of the hill. The height at the bottom of the hill is -10 feet. Find h .

Location	Verbal Description	Expression
Start	The height at the start is h .	h
Top of hill	The height at the top of the hill is 10 times the starting height h .	$10h$
Bottom of hill	Height decreases by 100 feet. So, subtract 100.	$10h - 100$

The height at the bottom of the hill is -10 feet. Solve $10h - 100 = -10$ to find h .

$$10h - 100 = -10 \quad \text{Write equation.}$$

$$10h = 90 \quad \text{Add 100 to each side.}$$

$$h = 9 \quad \text{Divide each side by 10.}$$

So, the height at the start is 9 feet.

On Your Own

Solve the equation. Check your solution.

7. $4 - 2y + 3 = -9$ 8. $7x - 10x = 15$ 9. $-8 = 1.3m - 2.1m$

10. **WHAT IF?** In Example 4, the height at the bottom of the hill is -5 feet. Find the height h .

Now You're Ready
Exercises 29–34



Vocabulary and Concept Check

1. **WRITING** How do you solve two-step equations?

Match the equation with the first step to solve it.

2. $4 + 4n = -12$ 3. $4n = -12$ 4. $\frac{n}{4} = -12$ 5. $\frac{n}{4} - 4 = -12$
- A. Add 4. B. Subtract 4. C. Multiply by 4. D. Divide by 4.



Practice and Problem Solving

Solve the equation. Check your solution.

- 1 6. $2v + 7 = 3$ 7. $4b + 3 = -9$ 8. $17 = 5k - 2$
9. $-6t - 7 = 17$ 10. $8n + 16.2 = 1.6$ 11. $-5g + 2.3 = -18.8$
12. $2t - 5 = -10$ 13. $-4p + 9 = -5$ 14. $11 = -5x - 2$
15. $4 + 2.2h = -3.7$ 16. $-4.8f + 6.4 = -8.48$ 17. $7.3y - 5.18 = -51.9$

ERROR ANALYSIS Describe and correct the error in finding the solution.

18.

$$\begin{array}{l} \times \quad -6 + 2x = -10 \\ -6 + \frac{2x}{2} = \frac{-10}{2} \\ -6 + x = -5 \\ x = 1 \end{array}$$

19.

$$\begin{array}{l} \times \quad -3x + 2 = -7 \\ -3x = -9 \\ \frac{-3x}{3} = \frac{-9}{3} \\ x = -3 \end{array}$$

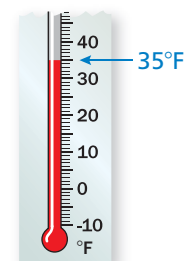
Solve the equation. Check your solution.

- 2 20. $\frac{3}{5}g - \frac{1}{3} = -\frac{10}{3}$ 21. $\frac{a}{4} - \frac{5}{6} = -\frac{1}{2}$ 22. $-\frac{1}{3} + 2z = -\frac{5}{6}$
23. $2 - \frac{b}{3} = -\frac{5}{2}$ 24. $-\frac{2}{3}x + \frac{3}{7} = \frac{1}{2}$ 25. $-\frac{9}{4}v + \frac{4}{5} = \frac{7}{8}$

In Exercises 26–28, write an equation. Then solve.

26. **WEATHER** Starting at 1:00 P.M., the temperature changes -4 degrees per hour. How long will it take to reach -1° ?
27. **BOWLING** It costs \$2.50 to rent bowling shoes. Each game costs \$2.25. You have \$9.25. How many games can you bowl?
28. **CELL PHONES** A cell phone company charges a monthly fee plus \$0.25 for each text message. The monthly fee is \$30.00 and you owe \$59.50. How many text messages did you have?

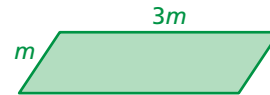
Temperature
at 1:00 P.M.



Solve the equation. Check your solution.

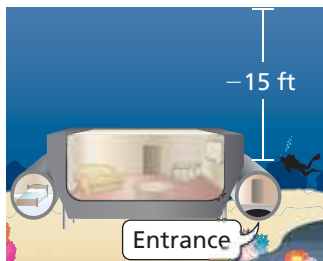
- 3 29. $3v - 9v = 30$ 30. $12t - 8t = -52$ 31. $-8d - 5d + 7d = 72$
 32. $6(x - 2) = -18$ 33. $-4(m + 3) = 24$ 34. $-8(y + 9) = -40$
 35. **WRITING** Write a real-world problem that can be modeled by $\frac{1}{2}x - 2 = 8$. Then solve the equation.

36. **GEOMETRY** The perimeter of the parallelogram is 102 feet. Find m .



REASONING Exercises 37 and 38 are missing information. Tell what information is needed to solve the problem.

37. **TAXI** A taxi service charges an initial fee plus \$1.80 per mile. How far can you travel for \$12?
 38. **EARTH** The coldest surface temperature on the moon is 57 degrees colder than twice the coldest surface temperature on Earth. What is the coldest surface temperature on Earth?
 39. **SCIENCE** On Saturday, you catch insects for your science class. Five of the insects escape. The remaining insects are divided into three groups to share in class. Each group has nine insects. How many insects did you catch on Saturday?
 a. Solve the problem by working backwards.
 b. Solve the equation $\frac{x - 5}{3} = 9$. How does the answer compare with the answer to part (a)?



40. **UNDERWATER HOTEL** You must scuba dive to the entrance of your room at Jule's Undersea Lodge in Key Largo, Florida. The diver is 1 foot deeper than $\frac{2}{3}$ of the elevation of the entrance. What is the elevation of the entrance?

41. **Geometry** How much should you change the length of the rectangle so that the perimeter is 54 centimeters? Write an equation that shows how you found your answer.



Fair Game Review What you learned in previous grades & lessons

Multiply or divide.

42. -6.2×5.6 43. $\frac{8}{3} \times \left(-2\frac{1}{2}\right)$ 44. $\frac{5}{2} \div \left(-\frac{4}{5}\right)$ 45. $-18.6 \div (-3)$
 46. **MULTIPLE CHOICE** Which fraction is *not* equivalent to 0.75?
 (A) $\frac{15}{20}$ (B) $\frac{9}{12}$ (C) $\frac{6}{9}$ (D) $\frac{3}{4}$